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It is now a matter of routine exercise to pass to the various limits as Ω, ω, $\mathscr{E}$, or combinations of them and obtain the different cases appearing in the literature.

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Absolute intensities of cosmic ray muons above 3·48 and 7·12 GeV/*c*

Abstract. The new Durham spectrograph MARS has been used to determine absolute intensities of cosmic ray muons in the near vertical direction with momenta above 3·48 and 7·12 GeV/*c*. The intensities are found to be close to those previously reported by Aurela and Wolfendale in 1967, the present intensities being higher by some (7·7 ± 1·3)% and (1·7 ± 1·4)%, at the respective momenta. Comparison is also made with the results of other recent measurements.

The momentum spectrum of cosmic ray muons at ground level is a key measurement not only for cosmic ray phenomenology but also because of its relevance to the energetic interactions in the upper levels of the atmosphere, from which the parents of the muons are derived, and to the interpretation of the subsequent behaviour of the muons in their penetration to great depths underground. Absolute intensities rather than spectral shapes are often of importance and where these have not been determined in experiments (usually because of difficulties concerning uncertain edge effects of detectors) it has been customary to normalize the results to a value given by Rossi (1948) at 1 GeV/*c*. This procedure has been followed by many workers but, recently, doubt has been cast on the validity of this standard intensity by the Kiel group (Allkofer *et al.* 1970, 1971, private communication) and in turn on those spectra which were normalized to the Rossi value.

The purpose of the present note is to explain the status of intensity measurements made with the previous Durham vertical spectrograph and to give absolute intensities above 3·48 and 7·12 GeV/c measured with the new instrument MARS (Ayre *et al.* 1969).

The derivation of the spectrum from a previous vertical instrument embodying an air gap magnet with neon flash tubes as detectors was given by Gardener *et al.* (1962) and Hayman and Wolfendale (1962, to be referred to as HW). Geiger counters were used for particle selection and normalization was made to Rossi's differential value at 1 GeV/c, momentum dependent bias effects introduced by the Geiger counter 'momentum selector' being adjudged to be small. In later work an attempt was made to calculate this bias and make allowance for it. Thus, Hayman *et al.* (1963) estimated that the intensities should be increased by a factor which varies from some 4% at 100 GeV to approximately 20% at 1000 GeV. Osborne *et al.* (1964) made further small corrections above 50 GeV and Aurela and Wolfendale (1967, to be referred to as AW) carried out a new experiment using a modified experimental arrangement to study the bias further. The result of the latter work is shown in figure 1, along with the original HW spectrum, for the momentum range $1 < p < 10$ GeV/c. Both are normalized to the *integral* intensity at 1 GeV/c ($5{\cdot}90 \times 10^{-3}$ cm^{-2} sr^{-1} s^{-1}) which comes from Rossi's spectrum.

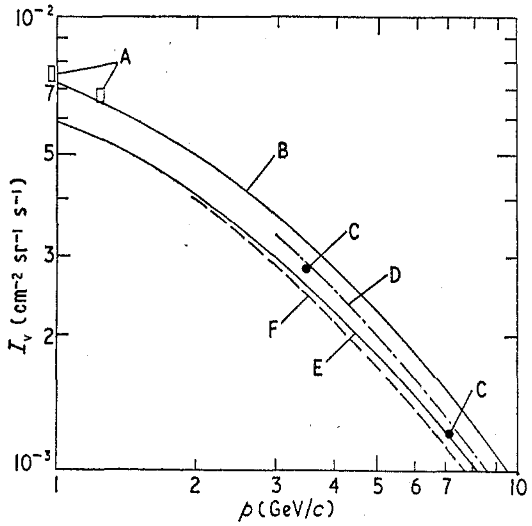

Figure 1. The integral energy spectrum of near vertical cosmic ray muons. A, Allkofer *et al.* (1970); B, Allkofer *et al.* (1971, private communication); C, present work; D, Duller *et al.* (1971, private communication); E, Aurela and Wolfendale (1967); F, Hayman and Wolfendale (1962).

MARS (magnetic automated research spectrograph) has been described in detail by Ayre *et al.* (1969). Briefly, it consists of four magnet blocks each 3·7 m × 2·1 m × 1·2 m arranged vertically. In the gaps between the blocks and also at the top and bottom of the instrument are trays of flash tubes used as detectors. The spectrograph is triggered by a threefold coincidence of scintillation counters placed at the top, centre and bottom of the instrument and the selected particles traverse the magnetized iron. The spectrograph is in two symmetrical halves and each half can be operated independently of the other.

Although it is intended to eventually digitize the flash tube trays, one half has been operated with photographic recording of the flash tube information to check the performance of the spectrograph. This same half has been used to measure the absolute intensity.

For the present experiment the instrument has been used under the condition of zero magnetic field, the counting rate being measured to high precision and the flash tube photographs being employed to determine what fraction of the events were not genuine. Measurements were made of the rate of particles penetrating the whole instrument ($p > 7{\cdot}12$ GeV/c) and those penetrating the upper half ($p > 3{\cdot}48$GeV/c). The collecting power (acceptance) of the spectrograph has been calculated by the method of Lovati *et al.* (1954) and allowance has been made for scattering effects, obliquity factors and inefficiency in the scintillation counters.

An estimate of the effect of scattering has been made using a Monte Carlo calculation. For both intensity measurements the effect was an upward correction of about 1%.

In calculating the acceptance of the instrument allowance is necessary for the incident angular distribution. Over the range of angles in question ($\phi < 29°$) the intensity varies as $I_V \cos^n\phi$, where n is a slowly varying function of momentum. Values of n for the respective median momenta have been taken from the work of Moroney and Parry (1954) and the calculated acceptances are as given in table 1.

Table 1. Basic data for absolute muon intensity measurements at Durham

Period of measurement: March–April, 1971.
Mean atmospheric pressure = 748·7 mm of Hg.

Minimum momentum (GeV/c)	Mean corrected threefold rate (min^{-1})	Acceptance (cm^{2}sr)	Integral intensity (cm^{-2}sr^{-1}s^{-1})
3·48	252·1 ±2·0	1507 ±12	$(2{\cdot}78 \pm 0{\cdot}036)10^{-3}$
7·12	30·74 ±0·24	422 ±3	$(1{\cdot}21 \pm 0{\cdot}017)10^{-3}$

The spectrograph is situated at an altitude of 61 m above mean sea level and the geographical coordinates are: latitude 54·5 °N and longitude 1·3 °W.

The angular distribution has also been used in calculating the effective mean path lengths in the magnetic block, and these path lengths have been corrected for the zig-zag motion of the particles, due to Coulomb scattering, by the method of Koenig (1946). The result is an increase in effective absorber thickness by 0·4% at the higher momentum and 0·8% at the lower. These mean path lengths have been converted to momentum thresholds using the calculations of Serre (1967), energy losses other than ionization being negligible. Straggling has been considered but the correction to the effective minimum range is quite negligible.

The efficiency of each scintillation counter is dependent on discriminator settings and it has been determined from an examination of the pulse height distribution from each pair of photomultipliers (two diagonally connected pairs of photomultipliers view each scintillator). In this test, the counted pulses were in coincidence with pulses from the other two scintillators. By subtracting the noise distribution of the pair from the pulse height distribution and superimposing the discriminator cut off levels the efficiency of the pair could be estimated. In this way the efficiency of each of the six pairs of photomultipliers was determined to an accuracy of about 0·5% and hence the overall efficiency to an accuracy of about 1·2%. The efficiencies were determined for three discriminators settings (covering the range of overall efficiency 89–96%) and the experimental rates were measured for these settings; the rates after correction were found to be in good agreement for the three settings. Mean rates are given in table 1.

Corrections are necessary for side showers and other events where a single muon did not penetrate the whole instrument. The photographic records were examined to determine these corrections.

The final vertical intensities determined from the data are shown in figure 1 and given in table 1 along with geographical and atmospheric data.

Comparison with the AW spectrum shows good agreement at 7·12 GeV/*c* (a difference of only (1·7 ± 1·4%)) but the present intensity at 3·48 GeV/*c* is higher by (7·7 ± 1·3%). Although the difference is within the 10% error on the normalization value quoted by Rossi (1948), it does appear as though the spectrum may be significantly steeper in this region than that given by AW. Accordingly the integral value at 1 GeV/*c* may well be higher than given by the Rossi value.

Figure 1 also shows intensities measured by the Kiel group. The two points near 1 GeV/*c* refer to absorption measurements and the full line is a best fit through absolute intensities from a variety of spectrographs (Allkofer *et al.* 1971, private communication). Our intensities are both significantly below this line.

The results of very recent measurements by Duller *et al.* (1971, private communication from Professor W. R. Sheldon) are also given in figure 1. It is noted that they are not inconsistent with the present measurements.

In conclusion, although the standard intensity at 1 GeV/*c* has not been checked (and may, indeed be significantly in error) the new measurements show that the intensities quoted by Aurela and Wolfendale (1967) are unlikely to be in error by more than 10%, in the momentum range 3–8 GeV/*c*.

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Nuclear charge distributions of ^{89}Y, ^{90}Zr and ^{92}Mo by elastic electron scattering†

Abstract. Elastic scattering of electrons has been used to extract the nuclear charge distribution parameters of the $N = 50$ isotones; ^{89}Y, ^{90}Zr and ^{92}Mo. The momentum transfer range studied is from 0·25 fm^{-1} to 1·15 fm^{-1}. A comparison of the differences in the charge distributions is made with the shell model predictions.

Nuclear charge distribution parameters have been obtained using elastic electron scattering in various laboratories (Hofstadter and Collard 1967). As a gross property of the elements in the periodic table, the charge root mean square radius follows an $A^{1/3}$ dependence. The study of isotopes has revealed significant departures from this behaviour (Hofstadter *et al.* 1965, Singhal *et al.* 1970, Curran *et al.* 1971), and it is of interest to investigate the corresponding variations for a group of isotones (Sinha *et al.* 1971). If the additional protons in such a group are in the vicinity of a closed shell, the differences between the charge distributions should reflect the spatial distribution of these protons. This will be particularly true if the isotones have a closed major neutron shell. In this letter we present the results of our study of the isotone triplet, ^{89}Y, ^{90}Zr and ^{92}Mo with $N = 50$.

The isotone targets were in the form of metal foils of isotopic enrichment greater than 98% and a graphite target was used as a reference standard. The measurements were made with the Glasgow electron scattering facility described by Hogg *et al.* (1971) and the details of the data analysis can be found in Singhal *et al.* (1971).

An harmonic oscillator charge distribution, which includes corrections for the centre of mass motion and the finite proton size, was used for ^{12}C. The charge distribution parameters were taken from Bentz *et al.* (1967) and they are $a = 1{\cdot}669$ fm and $\alpha = 1{\cdot}006$. For the isotones, a two parameter Fermi distribution characterized by a half density radius c and a surface thickness t was used (Elton 1961). A Rawitscher–Fischer phase shift code was used to calculate all the cross sections, and

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